Quotient Rule From Product Rule

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We want to use

Product Rule: $(f(x)g(x))' = f(x)g'(x) + f'(x)g(x)$

to differentiate $\frac{f(x)}{g(x)}$.

① Apply product rule to $f(x)\cdot\frac{1}{g(x)} = \frac{f(x)}{g(x)}$.

$$\left(\frac{f(x)}{g(x)}\right)' = \left(f(x)\cdot\frac{1}{g(x)}\right)' = f(x)\left(\frac{1}{g(x)}\right)' + f'(x)\frac{1}{g(x)}$$

② Find $\left(\frac{1}{g(x)}\right)'$: How to find using product rule?

TRICK: $g(x)\cdot\frac{1}{g(x)} = 1$

(product rule) → $g(x)\left(\frac{1}{g(x)}\right)' + g'(x)\frac{1}{g(x)} = 0$ ↓ Take $\frac{d}{dx}$

algebra: $g(x)\left(\frac{1}{g(x)}\right)' = -g'(x)\frac{1}{g(x)}$

$$\Rightarrow \left(\frac{1}{g(x)}\right)' = \frac{-g'(x)}{g(x)^2}$$

③ Put together ① & ②:

$$\left(\frac{f(x)}{g(x)}\right)' = f(x)\left(\frac{-g'(x)}{g(x)^2}\right) + f'(x)\frac{1}{g(x)}$$

(common denominator) → $= \frac{-f(x)g'(x)}{g(x)^2} + \frac{f'(x)g(x)}{g(x)^2}$

$\Rightarrow$ **Quotient Rule:** $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$