4/13/22 – Parametric Equations – Yigal Kamel

Looking back: Throughout the calculus sequence we have studied different types of functions (i.e. with different inputs & outputs).

	INPUT → OUTPUT	Ex:
Calculus I:	$f : \mathbb{R} \longrightarrow \mathbb{R}$, $x \longmapsto f(x)$; (real number) → (real number)	• $f(x) = x^2$ • $f(x) = \sin x$ • $f(x) = \begin{cases} 0, & x \leq 0 \\ x^3 e^x, & x \geq 0 \end{cases}$
Sequences:	$a : \mathbb{N} \longrightarrow \mathbb{R}$, $n \longmapsto a_n$; (positive integer) → (real number)	• $a_n = \frac{1}{n}$ • Series ↳ $\sum_{n=1}^{\infty} \frac{1}{n^2}$ ↳ $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$
Sequences of functions: [this example is a bit more obscure, so don't worry if it's confusing.]	$f : \mathbb{N} \times \mathbb{R} \longrightarrow \mathbb{R}$, $(n, x) \longmapsto f_n(x)$; (positive integer and a real number) → (real number)	• Power series ↳ $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ ↳ $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ (• Fourier Series) → haven't learned in this class

Now: Parametric equations are our way of studying a new kind of function.

Parametric equations: (a.k.a. "vector valued functions")

$$(x, y) : \mathbb{R} \longrightarrow \mathbb{R}^2$$

$$t \longmapsto (x(t), y(t))$$

(real number) → (pair of real numbers a.k.a. "vector")

[Note: We may also encounter examples with more than two outputs. The general case is $f: \mathbb{R} \longrightarrow \mathbb{R}^n$.]

Ex:
- $(x(t), y(t)) = (\cos t, \sin t)$
- the position of a particle traveling in a plane over time.
- any function $f(x)$ of the "Calc I" type, via $(x(t), y(t)) = (t, f(t))$
- any pair of functions of the "Calc I" type: $(x(t), y(t)) = (f(t), g(t))$.

There are a number of ways to think about, or describe, such a function. Two good ones to keep in mind are:

(1) as a pair of ordinary (Calc I) functions,

(2) as a point/vector in the plane that varies in "time" t.