

There are two things one can mean by the word "Integral".

(A) Definite Integral

- $\int_a^b f(x) dx :=$ limit of Riemann sums
= "area under the curve"

(B) Indefinite Integral

- $\int f(x) dx := F(x) + C$, where $F'(x) = f(x)$
i.e. a "general anti-derivative" of $f(x)$

These two notions are different a priori.

The fundamental theorem of calculus says they happen to be the same.

Theorem (F.T.C.): (A) \iff (B)

What this means is that knowing how to find one of the two items always enables you to find the other:

(A) \implies (B): $F(x) = \int_a^x f(t) dt$ is an anti-derivative for $f(x)$.

(B) \implies (A): $\int_a^b f(x) dx = F(b) - F(a)$, where $F(x)$ is an anti-derivative of $f(x)$.

Notice that these two statements are the content of FTC parts I & II.

So the FTC says exactly that the two notions of Integral carry the SAME information.