Integrals and the F.T.C. — Yigal Kamel

There are two things one can mean by the word "Integral".

(A) Definite Integral

- $\int_a^b f(x)dx :=$ limit of Riemann sums
  $=$ "area under the curve"

(B) Indefinite Integral

- $\int f(x)dx :=$ $F(x) + C$, where $F'(x) = f(x)$
  i.e. a "general anti-derivative" of $f(x)$

These two notions are different, a priori.
The fundamental theorem of calculus says they happen to be the same.

Theorem (F.T.C.): (A) $\Leftrightarrow$ (B)

What this means is that knowing how to find one of the two items always enables you to find the other:

(A) $\Rightarrow$ (B): $F(x) = \int_a^x f(t)dt$ is an anti-derivative for $f(x)$.

(B) $\Rightarrow$ (A): $\int_a^b f(x)dx = F(b) - F(a)$, where $F(x)$ is an anti-derivative of $f(x)$.

Notice that these two statements are the content of FTC parts I & II.

So the FTC says exactly that the two notions of Integral carry the SAME information.