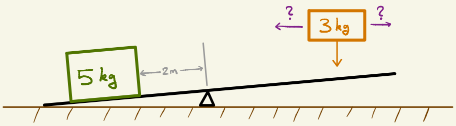
# 3/2/22 – CENTER OF MASS (IN ONE DIMENSION) – YIGAL KAMEL




we find a green 5kg weight on a seesaw. we want to place an orange 3kg weight on the opposite side of the fulcrum so that the seesaw balances.

$$\underline{\text{TORQUE}} = \underbrace{(\text{FORCE})}_{(\text{mass})\cdot g} \cdot (\text{DISTANCE FROM FULCRUM})$$

$$\Longrightarrow \text{want: } \quad 5\cdot g\cdot 2 = 3\cdot g\cdot d \;\Rightarrow\; \boxed{d = \frac{10}{3}}$$

where $d$ is the distance from the orange mass to the fulcrum.

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If we relable $\boxed{5\,kg}$ as $m_1$, $\boxed{3\,kg}$ as $m_2$, & view the seesaw as an $x$-axis with the fulcrum at zero, then the "balancing" eqn. is:

$$\boxed{m_1 x_1 + m_2 x_2 = 0}$$

where $x_i$ is the location of $m_i$ on the axis. For "$n$" masses, this extends to:

$$\boxed{\sum_{i=1}^{n} m_i x_i = m_1 x_1 + m_2 x_2 + \cdots + m_n x_n = 0}$$

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Q: What if we don't know where the "center of mass" (= fulcrum) is?

A: Give the unknown center of mass a name: $\bar{x}$. Then:

$$\sum_{i=1}^{n} m_i (x_i - \bar{x}) = 0 \iff \sum_{i=1}^{n} m_i x_i = \sum_{i=1}^{n} m_i \bar{x} = \left(\sum_{i=1}^{n} m_i\right)\bar{x}$$

[Notice: $m = \sum_{i=1}^{n} m_i$ is the total mass. So the center of mass is the location where we can treat all the mass as being located there.]

Dividing gives:

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{\text{total "moment"}}{\text{total mass}}}$$

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We do the analogous thing for a continuously distributed mass:

$$\boxed{\bar{x} = \frac{\int_I x\,dm}{\int_I dm}}$$

where $I$ is the interval over which the mass is defined.