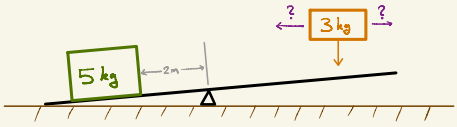
(DISCRETE) CENTER OF MASS (IN ONE DIMENSION) — YIGAL KAMEL



we find a green 5kg weight on a seesaw. we want to place an orange 3kg weight on the opposite side of the fulcrum so that the seesaw balances.

$$\text{TORQUE} = \underbrace{(\text{FORCE})}_{(\text{mass})\cdot g} \cdot (\text{DISTANCE FROM FULCRUM})$$

$$\Rightarrow \text{want: } 5\cdot g\cdot 2 = 3\cdot g\cdot d \Rightarrow \boxed{d = \frac{10}{3}}$$

where d is the distance from the orange mass to the fulcrum.

If we relable 5kg as m_1, 3kg as m_2, & view the seesaw as an x-axis with the fulcrum at zero, then the "balancing" eqn. is:

$$\boxed{m_1 x_1 + m_2 x_2 = 0}$$

where x_i is the location of m_i on the axis. For "n" masses, this extends to:

$$\boxed{\sum_{i=1}^{n} m_i x_i = m_1 x_1 + m_2 x_2 + \cdots + m_n x_n = 0}$$

Q: What if we <u>don't know</u> where the "center of mass" (= fulcrum) is?

A: Give the unknown center of mass a name: $\bar{x}$. Then:

$$\sum_{i=1}^{n} m_i (x_i - \bar{x}) = 0 \iff \sum_{i=1}^{n} m_i x_i = \sum_{i=1}^{n} m_i \bar{x} = \left(\sum_{i=1}^{n} m_i\right)\bar{x}$$

[Notice: $m = \sum_{i=1}^{n} m_i$ is the <u>total</u> mass. So the center of mass is the location where we can treat all the mass as being located there.]

Dividing gives:

$$\boxed{\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} = \frac{\text{total "moment"}}{\text{total mass}}}$$

We do the analogous thing for a continuously distributed mass:

$$\boxed{\bar{x} = \frac{\int_I x\, dm}{\int_I dm}}$$

where I is the interval over which the mass is defined.