Math 199, Fall 2022
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Preparation assignment 9 - Alternating series (and conditional convergence)

Estimated time: 30 minutes - 1 hour.

Point value: 2 points.

Goals: Appreciate the subtleties of working with series which are conditionally convergent.

For this worksheet, we'll consider the **alternating harmonic series**: $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$.

This is like the harmonic series, but the signs of the terms alternate between positive and negative.

1) Compute the first few partial sums:

(a) $s_1 = \displaystyle\sum_{n=1}^{1}(-1)^{n+1}\frac{1}{n} = (-1)^{1+1}\frac{1}{1} =$

(b) $s_2 = \displaystyle\sum_{n=1}^{2}(-1)^{n+1}\frac{1}{n} = (-1)^{1+1}\frac{1}{1} + (-1)^{2+1}\frac{1}{2} =$

(c) $s_3 = \displaystyle\sum_{n=1}^{3}(-1)^{n+1}\frac{1}{n} =$

(d) $s_4 =$

2) Draw an axis and label the points 0 and 1. Plot and label the first six partial sums "s_n" on your axis.

3) Does it look like the series will converge or diverge? If you said it will converge, describe a small interval $[a, b]$ which you think the sum lands in. Justify your answer.

Returning to the alternating harmonic series, writing out the beginning of the sum looks like:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} - \frac{1}{10} + \frac{1}{11} - \frac{1}{12} + \frac{1}{13} - \frac{1}{14} + \frac{1}{15} - \cdots$$

4) Let's rearrange the same terms of the sum into the pattern "odd, odd, even, odd odd, even, ..." like:

$$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \frac{1}{13} + \frac{1}{15} - \frac{1}{8} + \cdots$$

Group together the terms above into "periods of length 3" (i.e. the signs within a period should look like "$+, +, -$"). Explain why each of these periods sum to a *positive* number.

5) Try to find a lower bound for the sum in problem (4). Compare this to your answer to problem (3).

6) *Bonus (challenge):* Rearrange the terms of the alternating harmonic series so that the resulting series diverges. (You don't have to be super explicit about your construction.)