Math 199, Spring 2022
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Preparation Assignment 8 - Alternating series (and conditional convergence)

Estimated Time: 30 minutes - 1 hour.

Goals: Our aim on Friday will be to understand the convergence properties of alternating series and appreciate the subtleties of working with series which are conditionally convergent.

For this worksheet, we'll consider the **alternating harmonic series**: $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$.

This is like the harmonic series, but the signs of the terms alternate between positive and negative.

1) Compute the first few partial sums:

(a) $s_1 = \sum_{n=1}^{1}(-1)^{n+1}\frac{1}{n} = (-1)^{1+1}\frac{1}{1} =$

(b) $s_2 = \sum_{n=1}^{2}(-1)^{n+1}\frac{1}{n} = (-1)^{1+1}\frac{1}{1} + (-1)^{2+1}\frac{1}{2} =$

(c) $s_3 = \sum_{n=1}^{3}(-1)^{n+1}\frac{1}{n} =$

(d) $s_4 =$

2) Draw an axis and label the points 0 and 1. Plot and label the first six partial sums "s_n" on your axis.

3) Does it look like the series will converge or diverge? If you said it will converge, describe a small interval $[a, b]$ which you think the sum lands in. Justify your answer.

Returning to the alternating harmonic series, writing out the beginning of the sum looks like:

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \frac{1}{7} - \frac{1}{8} + \frac{1}{9} - \frac{1}{10} + \frac{1}{11} - \frac{1}{12} + \frac{1}{13} - \frac{1}{14} + \frac{1}{15} - \cdots$$

4) Let's rearrange the same terms of the sum into the pattern "odd, odd, even, odd odd, even, ..." like:

$$1 + \frac{1}{3} - \frac{1}{2} + \frac{1}{5} + \frac{1}{7} - \frac{1}{4} + \frac{1}{9} + \frac{1}{11} - \frac{1}{6} + \frac{1}{13} + \frac{1}{15} - \frac{1}{8} + \cdots$$

Group together the terms above into "periods of length 3" (i.e. the signs within a period should look like "$+, +, -$"). Explain why each of these periods sum to a *positive* number.

5) Try to find a lower bound for the sum in problem (4). Compare this to your answer to problem (3).

6) *Bonus (challenge):* Rearrange the terms of the alternating harmonic series so that the resulting series diverges. (You don't have to be super explicit about your construction.)