

Preparation assignment 8 - Sequences and series

Estimated time: 30 minutes - 1 hour.

Point value: 2 points.

Goals: We are about to shift gears into the second big topic of the course: sequences and series. We want to use this time to get our minds used to these new ideas.

- 1) Write down the definition of a(n) (infinite) sequence. How do you think about a sequence?
- 2) We are primarily interested in sequences whose terms get successively closer to some special point called the **limit** of the sequence. In this case, we say that the sequence **converges** to the limit. If there is no such special point, we say the sequence **diverges**. Try to write down a precise definition for what it means for a sequence $\{a_n\}_{n=1}^{\infty}$ to converge to the limit L .
- 3) Write down an example of a sequence that converges to 0.

We say a sequence $\{a_n\}_{n=1}^{\infty}$ is **bounded** if it's possible to find fixed numbers m and M , such that $m < a_n < M$ for every n .

- 4) Is it possible for an unbounded sequence to converge? If yes, give an example; if not, explain why not.
- 5) Is it possible for a bounded sequence to diverge? If yes, give an example; if not, explain why not.
- 6) Write down the definition of a(n) (infinite) series. How do you think about a series?
- 7) Explain why a series is just an example of a sequence. (Thus, we can apply the same notions of convergence and divergence.)
- 8) Write down an example of a series that converges. How do you know it converges?
- 9) If a sequence $\{a_n\}_{n=1}^{\infty}$ diverges, is it possible for the series $\sum_{n=1}^{\infty} a_n$ to converge? If yes, give an example; if not, explain why not.
- 10) If a sequence $\{a_n\}_{n=1}^{\infty}$ converges, is it possible for the series $\sum_{n=1}^{\infty} a_n$ to diverge? If yes, give an example; if not, explain why not.