Math 199, Spring 2022
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**Preparation Assignment 6 - Applications of integrals: the mass of the sun**

**Estimated Time:** 30-45 minutes (or substantially more if you are ambitious and do the bonuses).

**Goals:** We want to get comfortable with using integrals to compute physical quantities. This might be difficult. Applied situations are often more confusing than theoretical ones. That being said, it is good to practice with them. Do your best, and we'll talk more about it on Friday.

We want to approximate the mass of the sun. Due to the nature of the force of gravity, the density of (a portion of) the sun depends on the distance from the sun's center. A simple approximation for the density of the sun as a function of the distance $r$ from the center might be:

$$\rho(r) = \frac{250}{37}\left(149^{2-2r} - 1\right)$$

kilograms per cubic meter, where $r$ is measured in units where the radius of the sun is 1.
(This approximation is only reasonable when $r$ is between 0 and 1; $r = 0$ corresponds to the center and $r = 1$ corresponds to the outer surface.)

Recall that if an object has constant density $\rho$ and volume $V$, then

$$\text{Mass} = m = \rho V.$$

Since the density of the sun is not constant, we will have to approximate the mass locally in regions with approximately constant density, and then integrate. We will assume the shape of the sun is a ball ( = the inside of a sphere).

1) Draw a sketch of the sun, together with an $r$-axis. Indicate in your sketch an arbitrary value of $r$ between 0 and 1.

2) On the same sketch, indicate the region of the sun for which $\rho(r)$ is a good approximation for the density (corresponding to the interval $[r, r + dr]$). Describe the region.

3) Find a local estimate for the volume you described in (2). It should look something like $dV = f(r)dr$.

4) Use the formula for the mass given above and your answer to (3) to write down a local estimate for the mass corresponding to the interval $[r, r + dr]$.

5) Write down the definite integral that (approximately) computes the mass of the sun (don't compute).

6) Explain how you could compute the integral above (you do know how).

*Bonus:* Either compute the integral, or use a calculator to do it. Notice that due to our chosen units of $r$, your answer to (3) was not in cubic meters. Using the fact that the sun's radius is $696.34 \times 10^6$ meters, adjust the value you got for the integral to find the mass of the sun in kilograms.

*Bonus (difficult):* How did I come up with the function $\rho(r)$? How would you come up with one?