

Preparation assignment 5 - Areas of infinite regions

Estimated time: 30-45 minutes.

Point value: 2 points.

Goals: The goal of this assignment is to begin to think about how and when an “infinite region” can have a finite area. This leads to the notion of an improper integral.

Do the following on a separate sheet of paper.

1) Consider the function $f(x) = \frac{1}{x}$.

(a) Sketch the graph of f .

(b) Describe the region whose area is $\int_1^a f(x)dx$. Indicate this region on your sketch from (a).

(c) Sketch the regions whose areas are represented by the following integrals (ideally on the same graph). Then calculate the integrals.

$$\int_1^{10} f(x)dx =$$

$$\int_1^{100} f(x)dx =$$

$$\int_1^{1000} f(x)dx =$$

(d) As you increase the upper bound “ a ”, as in (b), what does the integral tend towards?

(e) The region below the graph of $f(x)$, above the x -axis, and to the right of the line $x = 1$ is “infinite” (or unbounded) in its extent. If you had to define the area of this region, what would you define it to be?

2) Repeat parts (c), (d) and (e) for the function $f(x) = \frac{1}{x^2}$.

3) If you were developing the theory yourself, what notation would you use for the area described in part (e) for a general (positive) function $f(x)$?

4) Reflect on your findings, and write down anything you think is interesting.