Math 199, Fall 2023
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Preparation assignment 2 - Understanding derivatives

Estimated time: 20-40 minutes.

Point value: 2 points.

Goals: Reflect on the definition and the idea behind derivatives. Think about what happens when we take derivatives of derivatives.

Take out a separate sheet of paper, and write responses to the following questions.

1) Watch this video by 3blue1brown through time 9:43. Use the perspective portrayed here to explain how you can use the derivative $f'(a)$ to give a simple approximation of the function $f(x)$ when x is close to a. Which features of $f(x)$ does this approximation preserve, and which features might be lost?

2) Finish watching the video. What is your initial reaction to the "paradox" about whether or not the car is moving?

3) Let's try to better understand the situation being dubbed a paradox. Recall that the function of the car's motion is $s(t) = t^3$, and its derivative was shown to be $s'(t) = 3t^2$. Now since $s'(t)$ is itself a function which depends on t, it has further derivatives $s''(t)$ and $s'''(t)$ (and so on). Calculate these.

4) The video observed that $s'(0) = 0$ to try to answer whether or not the car is moving at time $t = 0$. Evaluate $s''(0)$ and $s'''(0)$ and try to use these results to address the "paradox". Revisit your answer to question (1) and try to refine your answer about what information each successive derivative contains.

5) Reflect on what you learned by watching the video and doing this activity. Write down anything you think is interesting.

6) *Bonus:* Suppose $f(x)$ is a function whose successive derivatives all exist. If we knew all the derivatives $f'(a), f''(a), f'''(a), ...$ of a function $f(x)$ at a point $x = a$, do you think that is enough information to recover *everything* about $f(x)$ for *all* x? Explain your thoughts.