

Preparation assignment 2 - Trigonometric substitution

Estimated time: Less than 30 minutes.

Goals: The main goal of this assignment is to understand the geometric ideas that lie beneath the integration method of trigonometric substitution, or “trig sub”.

Point value: 2 points.

Reminder: You will be graded on your engagement with these preparation assignments. I am not looking for “right” or “wrong” answers. I am asking you to think about these ideas before class, and put something on paper. It is up to you how long you want to spend on this.

Take out a separate sheet of paper, and do the following.

1) Consider the integral:

$$\int \sqrt{1-x^2} dx$$

Think about how you might want to compute this integral. Write down a few possibilities, and briefly follow through with your ideas.

2) You might think that you should try some kind of substitution, and that’s a good idea, but the typical “ u -substitution” way of thinking from Calc 1 might leave you stumped.

The “ u -substitution” paradigm generally has you think quite algebraically; you look at the integrand, try to break it up into convenient pieces using your knowledge of derivative rules, and then piece the expression back together. For this type of integral, we want to think *geometrically*, and let that analysis inform our choice of a good substitution to make.

When you see the expression $\sqrt{1-x^2}$ (i.e. the expression we want to integrate), what two shapes come to mind? (*Hint: the title of this worksheet.*)

3) Draw a picture of a right triangle. Label the length of one of the legs as x and the hypotenuse as length 1. Then find the length of the other leg.

4) Using your knowledge of the unit circle and trigonometry, interpret your drawing in (3) as lying inside the unit circle in the plane, and re-label your legs in terms of the sine and cosine of an angle, θ .

5) (*Bonus:*) Can you use this idea to compute the original integral?