

Math 199, Spring 2022
Yigal Kamel
1/28/22

Preparation Assignment 2 - Trigonometric substitution

Estimated Time: Less than 30 minutes.

Goals: The main goal of this assignment is to understand the geometric ideas that lie beneath the integration method of trigonometric substitution, or “trig sub”.

1) If you haven’t already, try to complete all five ways of computing the area of the region from Participation Assignment 1 (1/26/22).

In that problem, there was an integral that popped up, in more than one part, that you might not have known how to compute; namely:

$$\int \sqrt{1-x^2} dx.$$

You might think that you should try some kind of substitution, and that’s a good idea, but the typical “ u -substitution” way of thinking from Calc 1 might leave you stumped.

The “ u -substitution” paradigm generally has you think quite algebraically; you look at the integrand, try to break it up into convenient pieces using your knowledge of derivative rules, and then piece the expression back together. For this type of integral, we want to think *geometrically*, and let that analysis inform our choice of a good substitution to make.

2) When you see the expression $\sqrt{1-x^2}$ (i.e. the expression we want to integrate), what two shapes come to mind? (*Hint: the title of this worksheet.*)

3) Draw a picture of a right triangle. Label the length of one of the legs as x and the hypotenuse as length 1. Then find the length of the other leg.

4) Using your knowledge of the unit circle and trigonometry, interpret your drawing in (3) as lying inside the unit circle in the plane, and re-label your legs in terms of sine and cosine of an angle, θ .

Bonus: Can you use this idea to compute the original integral?