Math 199, Spring 2022
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Preparation Assignment 13 - Polar coordinates

Estimated Time: 30 minutes - 1 hour.

Goals: Get acquainted with polar coordinates.

Recall that polar coordinates (r, θ) for a point P in the plane are given by:

- $r =$ the distance from the origin to P,
- $\theta =$ the (counter-clockwise) angle from the positive x-axis to the ray from the origin to P.

1) Suppose P is a point in the plane with polar coordinates (r, θ).

(a) Is there a number $\tilde{\theta} \neq \theta$, such that $(r, \tilde{\theta})$ are also polar coordinates for P? If so, what values of $\tilde{\theta}$ allow this? If not, why not?

(b) Is there a number $\tilde{r} \neq r$, such that $(\tilde{r}, \theta)$ are also polar coordinates for P? If so, what values of $\tilde{r}$ allow this? If not, why not?

(c) Are there numbers $\tilde{r} \neq r$ and $\tilde{\theta} \neq \theta$, such that $(\tilde{r}, \tilde{\theta})$ are also polar coordinates for P? If so, what values of $\tilde{r}$ and $\tilde{\theta}$ allow this? If not, why not?

(d) *Bonus:* Is it *ever* possible for $(r, \theta + 1)$ to represent the same point $P = (r, \theta)$?

2) Given an ordinary function $r = f(\theta)$, describe how you would sketch the function by interpreting (r, θ) as polar coordinates. Do the resulting graphs satisfy a property analogous to the "vertical line test"?

3) Try sketching the function $r = \theta$ in polar coordinates, for $0 \leq \theta \leq 4\pi$. Explain why graphing a function in polar coordinates as in (2) is more analogous to a parametric curve, rather than a function $y = f(x)$.

4) In fact, given $r = f(\theta)$, you can always define parametric equations $(x(t), y(t))$ that trace out the polar curve (r, θ). Describe how to do this.

(Note: This sounds fancy, but this is really just about knowing how to convert polar coordinates to rectangular coordinates. The reason I stated the problem this way is to help you recognize that polar curves are simply *examples* of parametric curves, rather than a new thing entirely.)