Math 199, Spring 2022
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Preparation Assignment 10 - Towards Taylor series

Estimated Time: 30-45 minutes.

Goals: The goal of this assignment is to understand the relationship between derivatives of a function and the terms of its corresponding power series.

Consider a (smooth) function $f(x)$.

1) Find an equation for the tangent line to f at $x = 0$ in the following way: Write down an expression for an arbitrary line

$$y = a_0 + a_1 x,$$

and find the constants a_0, a_1 by imposing that the equation should represent the tangent line to f at $x = 0$ (in other words: the values and slopes of y and f should agree at $x = 0$). Rewrite the equation for the line in terms of your (now known) values of a_0 and a_1.

2) In a similar fashion to what we did above, let's find a second order polynomial,

$$g_2(x) = a_0 + a_1 x + a_2 x^2,$$

whose second derivative ("curvature") agrees with that of f at $x = 0$, in addition to its value and slope (as before). In other words, find a_2, such that $g_2''(0) = f''(0)$.

Bonus: Can we use the same a_0 and a_1? Why?

3) Find a polynomial of degree n,

$$g_n(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n,$$

whose first n derivatives agree with those of f at $x = 0$, i.e.

$$g_n^{(k)}(0) = f^{(k)}(0)$$

for all $0 \leq k \leq n$.

4) Find a power series $g(x) = \sum_{n=0}^{\infty} a_n x^n$, for which $g^{(k)}(0) = f^{(k)}(0)$ for all k.

5) Give an example of a function f for which $g = f$.

6) *(Bonus)* Can you find a (smooth) function f for which g is not equal to f? You may use any resource you'd like to find an answer to this.