Math 199, Spring 2022
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Participation Assignment 9 - Absolute and conditional convergence

Estimated Time: 1 hour.

Goals: Understand the notions of absolute and conditional convergence. Practice recognizing whether a series converges absolutely, conditionally or diverges. Looking ahead to series of functions.

1) Determine whether the following series converge conditionally, absolutely, or diverge. Justify your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n\pi)}{n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n\pi + \frac{\pi}{2})}{n}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(3n\pi)}{n^2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos((4n+1)\pi)}{n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(n)}{n^3}$

(f) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ (for any number x)

(g) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ (for any number x)

(h) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ (for any number x)

2) Parts (f), (g), and (h) of problem (1) are examples of series of *functions* (as the terms depend on a variable "x"). These turn out to be familiar functions, and you might be able to guess what they are. Take a few derivatives of each of these series (term by term), and see if you can recognize the functions based on the patterns you see. *Hint: You'll need to compare the derivatives of (f) and (g) to figure those out. (h) is the easier one to start with.*

3) *(Bonus)* Derive an equation relating all three functions in (f), (g), and (h).
Hint: You may need to exploit the imaginary unit $i = \sqrt{-1}$.
Extra hint: This is quite a famous equation named after quite a famous mathematician.