Math 199, Fall 2023
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Participation assignment 9 - Optimization

Estimated time: 40-55 minutes.

Point value: 3 points.

Goals: Understand how to apply our knowledge of derivatives to optimize desired values.

1) *Relating the sizes of regions and their boundaries.*

(a) Find the side lengths of a rectangle with the smallest possible perimeter, given that the area is 16. What is this smallest possible perimeter?

(b) Find the side lengths of a rectangle with the largest possible area, given that the perimeter is 16. What is this largest possible area?

(c) Explain a relationship between parts (a) and (b). Also explain a general rule for minimizing perimeter or maximizing area for any rectangle.

(d) You are building a pool in an "oval" shape constructed by replacing two opposite sides of a rectangle with semicircles. You only have enough material for the perimeter to be 30 meters or less. How big can you make the pool (in terms of area)? What will it's dimensions/shape be?

(2) Given a quadratic equation $y = ax^2 + bx + c$, you probably learned in high school what shape the graph is.

(a) Draw a rough sketch (divide into two cases depending on the sign of a).

(b) In order to obtain a more accurate sketch of the function, it would help to plot a few points, preferably some notable ones. A good place to start is the bottom/top point. Find the x and y values of this point in terms of a general a, b, and c.

(c) Apply (b) to (more accurately) sketch the graph of $y = 3x^2 - 12x + 9$. Include labels of the bottom/top point, and the roots.

(3) Suppose your friends live in a house located near the bend of a curving road shaped like the graph of $y = x^2 + x - 1$. Relative to this graph, their front door is located at the origin. They'd like to put up a mailbox on the road at the spot closest to the front door, because they don't like walking far in the cold. Do them a favor and calculate where the mailbox should go. How far will it be from the door (units = decameters)?