Math 199, Fall 2023
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**Participation assignment 8 - Linear approximation**

**Estimated time:** 30-45 minutes.

**Point value**: 3 points.

**Goals:** Learn how to approximate functions by linear functions using derivatives. Practice using this technique in examples. Learn how to apply these ideas to track propagating errors in measurements.

1) For each of the following functions and values of $x_0$ and $\Delta x$,

i. find $\Delta f = f(x_0 + \Delta x) - f(x_0)$,

ii. calculate the linear estimate $df = f'(x_0)\Delta x$,

iii. deduce the error of the approximation $|\Delta f - df|$.

(a) $f(x) = x^2 + 2x$, $x_0 = 1$, $\Delta x = 0.1$.

(b) $f(x) = 2x^2 + 4x - 3$, $x_0 = -1$, $\Delta x = 0.1$.

(c) $f(x) = x^3 - x$, $x_0 = 1$, $\Delta x = 0.1$.

(d) $f(x) = x^4$, $x_0 = 1$, $\Delta x = 0.1$.

(e) $f(x) = x^{-1}$, $x_0 = 0.5$, $\Delta x = 0.1$.

2) *Tolerance in measurement.* We often have to deal with the fact that no measuring device is perfect. It's important to keep track of how wrong our values might be.

(a) Let's say you have a ruler which is at most 2% inaccurate. You measure the radius of a circle and use it to calculate the circle's circumference and area. Find the maximum percent error in your values for the circumference and area.

(b) In the previous example, suppose you were building a specific item involving a circle, and you needed the sizes of both the circumference and area to be accurate within 1%. You need to find a ruler which will let you achieve this. What tolerance must your ruler have? (i.e. find the maximum error the ruler can have so that you achieve accurate enough measurements of circumference and area.)