Math 199, Fall 2022
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Participation assignment 8 - Computing physical quantities

Estimated time: Less than 1 hour.

Point value: 3 points.

Goals: Practice using integrals to compute physical quantities.

A list of physical quantities and formulas for them that you may need to reference:

- **Mass:** the mass of an object of density ρ and volume V is
$$m = \rho V.$$
- **Work:** the work done when applying a (constant) force F to displace an item by d is
$$W = Fd.$$
- **Kinetic energy:** the kinetic energy of a particle of mass m moving at speed v is
$$K = \frac{1}{2}mv^2.$$
- **Electric field:** The electric field at a point in (1-dimensional) space P produced by a particle with charge q is given by Coulomb's law:
$$E = \frac{kq}{r^2},$$
where k is a constant of nature, and r is the distance between the particle and the point P.
- **Center of mass:** $(\bar{x}, \bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$.

1) You probably (intuitively) know that a force is required to compress a (metal) spring (i.e. shorten its length); think about replacing the batteries of some device. Hooke's law models this force as
$$F = kx,$$
where k is a constant (characteristic of the spring), and x is the amount the spring is compressed (or stretched), i.e. the change in the spring's length. Find the work required to compress a spring with $k = 9$ (Newtons per meter) from a resting length of 1 meter to a compressed length of 0.5 meters.

2) You happen upon a line segment of particles between the points $(-1, 1)$ and $(4, 2)$ which has total charge $q = 10$ (in some irrelevant unit system). Calculate the electric field you would feel (in these fake units) if you were standing at the point $(9, 3)$. *Bonus:* what if you were standing at the origin? (*Warning: the formula I gave you for the electric field must be interpreted differently for this.*)

3) You find a penny on the table and decide to spin it about one of its diameters (like a top). Suppose we know that the penny completes five full revolutions (i.e. $5 \cdot 2\pi$ radians) every second. Find the kinetic energy of the penny, where

- the penny is assumed to have no (or constant) thickness,
- the density of the penny is $\dfrac{5}{2\pi}$ grams per square centimeter,
- the diameter of the penny is 2 centimeters.

4) Find the center of mass of a thin triangular metal plate in the plane whose vertices are at $(0, 0)$, $(3, 0)$, and $(2, 1)$, where the density of the metal is the constant value $\rho = 4$.