Merit 231 WS 15

Sophie Larsen

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Comparison Test

Problem 1

Let $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ be series, with $a_n, b_n \geq 0$ for all n and $a_n \leq b_n$ for all n. Circle the correct answer for each of the following statements:

- If $\sum b_n$ is convergent, then $\sum a_n$ is (convergent/divergent/not enough information)
- If $\sum b_n$ is divergent, then $\sum a_n$ is (convergent/divergent/not enough information)
- If $\sum a_n$ is convergent, then $\sum b_n$ is (convergent/divergent/not enough information)
- If $\sum a_n$ is convergent, then $\sum b_n$ is (convergent/divergent/not enough information)

Choose two statements from this list and explain using a picture or diagram why the statement is true or false.

Problem 2

Consider the series $\sum_{n=2}^{\infty} \frac{n^2}{n^3-1}$.

(a)

First, try the Divergence Test on this series. What does it tell you? Can you conclude anything about divergence or convergence from this test?

(b)

Now, try applying the Comparison Test. What do you find? (Hint: look at the ratio of the highest terms in the numerator and denominator.)

(c)

How does your finding under the Comparison Test compare to your finding under the Divergence Test? Are the two findings consistent? Explain.

Problem 3

Determine the convergence of $\sum_{n=1}^{\infty} \frac{\cos(n)}{n^3}$.

Problem 4

Determine the convergence of $\sum_{n=0}^{\infty} \frac{2^n \sin^2(5n)}{4^n + \cos^2(n)}$