Math 199, Fall 2023
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Participation assignment 7 - Related rates

Estimated time: 45-60 minutes.

Point value: 3 points.

Goals: Learn how to break down a “related rates” problem step by step, i.e. how to compute the derivative of something in terms of the derivative of a related thing.

Related rates problems often start out with word problems. Let’s start out without the “words” to get used to the underlying math involved.

1) Let $h(x)$ be a differentiable function that we don’t have a formula for, let $g(x) = 3x^2 + h(x)$, and let $f(x) = h(x)^2$. Some way or another, we are able to figure out that $g(0) = 3$ and $g'(0) = 5$. Follow the steps below to find $f'(0)$.

(a) Use your knowledge of $g(0)$ to find $h(0)$.

(b) Find formulas for $g'(x)$ and $f'(x)$ in terms of $x, h(x)$, and/or $h'(x)$.

(c) Use your knowledge of $g'(0)$ and your formula for $g'(x)$ to find $h'(0)$.

(d) Put together your knowledge of $h(0), h'(0)$, and $f'(x)$ to find $f'(0)$ (as an actual number!).

2) Let's try that again without giving you broken down steps. Let $g(x) = p(x) - xq(x)$, $f(x) = p(x)q(x)$, and suppose $g(0) = 2$, $g'(0) = 0$, $q(0) = 3$, and $q'(0) = 5$. Find $f'(0)$.

3) The *kinetic energy* of an object is given by $KE = \frac{1}{2}mv^2$, where m is the mass of the object and v is its speed. A 1000 kilogram car is merging onto the highway going 20 meters per second and accelerating at 3 meters per second per second. In addition to its own weight, the car is also carrying 100 kilograms of sand, but it's leaking about 0.5 kg per second. Find the rate of change of the kinetic energy at this time (i.e. the *power* being exerted).

4) Let x, y, z be the edge lengths of a rectangular box, and suppose they are changing over time at the rates:

$$\frac{dx}{dt} = 1, \qquad \frac{dy}{dt} = -2, \qquad \frac{dz}{dt} = 1.$$

Find the rates at which the following quantities of the box are changing over time when $x = 4, y = 3, z = 2$:

(a) volume;

(b) surface area;

(c) diagonal length between opposite corners.

5) *A growing raindrop.* A spherical drop of mist is growing by picking up moisture from the air via condensation. Suppose the rate at which the drop picks up water is proportional to its surface area. Show that the radius of the drop is growing at a *constant* rate.