Math 199, Spring 2022
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Participation Assignment 7 - Applications of integrals: computing physical quantities

Estimated Time: Less than 1 hour.

Goals: Practice using integrals to compute physical quantities.

For this worksheet, if time is running short, it's okay to just set up the intergals and move on.

A list of physical quantities and formulas for them that you may need to reference:

- **Mass:** the mass of an object of density ρ and volume V is
$$m = \rho V.$$
- **Work:** the work done when applying a (constant) force F to displace an item by d is
$$W = Fd.$$
- **Kinetic energy:** the kinetic energy of a particle of mass m moving at speed v is
$$K = \frac{1}{2}mv^2.$$
- **Electric field:** The electric field at a point in (1-dimensional) space P produced by a particle with charge q is given by Coulomb's law:
$$E = \frac{kq}{r^2},$$
where k is a constant of nature, and r is the distance between the particle and the point P.
- **Center of mass:** we did this on the blackboard.

1) You probably (intuitively) know that a force is required to compress a (metal) spring (i.e. shorten its length); think about replacing the batteries of some device. Hooke's law models this force as

$$F = kx,$$

where k is a constant (characteristic of the spring), and x is the amount the spring is compressed (or stretched), i.e. the change in the spring's length. Find the work required to compress a spring with $k = 9$ (Newtons per meter) from a resting length of 1 meter to a compressed length of 0.5 meters.

2)* You happen upon a line segment of particles between the points $(-1, 1)$ and $(4, 2)$ which has total charge $q = 10$ (in some irrelevant unit system). Calculate the electric field you would feel (in these fake units) if you were standing at the origin. *Bonus:* what if you were standing at the point $(-2, 1)$?

*After class correction: While using the formula I gave you for the electric field above for this problem is still good practice with setting up integrals (there are no internal mathematical issues), it is not correct physically. The formula above is only correct for a 1-dimensional setting, while for this situation, the 2-dimensional version is needed to get the real (physical) answer. This requires interpreting the electric field as a *vector* instead of just a number.

3) You find a penny on the table and decide to spin it about one of its diameters (like a top). Suppose we know that the penny completes five full revolutions (i.e. $5 \cdot 2\pi$ radians) every second. Find the kinetic energy of the penny, where

- the penny is assumed to have no (or constant) thickness,
- the density of the penny is $\dfrac{5}{2\pi}$ grams per square centimeter,
- the diameter of the penny is 2 centimeters.

4) Find the center of mass of a thin triangular metal plate in the plane whose vertices are at $(0, 0)$, $(3, 0)$, and $(2, 1)$, where the density of the metal is the constant value $\rho = 4$.