Math 199, Fall 2022
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Participation assignment 5 - Improper integrals

Estimated time: 1 hour.

Point value: 3 points.

Goals: We want to understand what improper integrals are, how to recognize them, and how to deal with them.

1) What are the two different types of improper integrals?

2) Consider the function $f(x) = \dfrac{1}{x^2}$.
(a) True or false:

$$\int_{-1}^{1} \frac{1}{x^2} dx = -\frac{1}{x}\Big|_{-1}^{1} = -1 - 1 = -2.$$

(b) Notice that the function $\dfrac{1}{x^2}$ is always positive where it is defined, but the answer on the right side of the above equation is negative. Is this a problem? Explain why or why not.

(c) Calculate $\int_0^1 \frac{1}{x^2} dx$, or show that it diverges.

(d) Calculate $\int_{-1}^0 \frac{1}{x^2} dx$, or show that it diverges.

(e) Do your answers satisfy $\int_{-1}^0 \frac{1}{x^2} dx + \int_0^1 \frac{1}{x^2} dx = -2$?

(f) Explain the collective results for this problem thus far. If your answer to (a) is false, explain the correct way to handle the problem.

3) This problem is a reflection on problem (2). If you prefer to do more practice, feel free to move to problem 4 and return to this after. For each of the following integrals, evaluate or show that it diverges.

(a) $\int_1^\infty \frac{1}{x^2} dx$,

(b) $\int_0^1 \frac{1}{x^2} dx$, (you did this already),

(c) $\int_1^\infty \frac{1}{\sqrt{x}} dx$,

(d) $\int_0^1 \frac{1}{\sqrt{x}} dx$,

Describe a precise relationship between parts (a) and (d) of this problem. Do the same for parts (b) and (c). (*Hint:* turn your head 90 degrees, or interchange the x and y axes.)

4) Determine whether the following improper integrals converge or diverge. Explain why. You do *not* need to find the values of the integrals.

(a) $\int_1^\infty 5 dx$.

(b) $\int_4^\infty \frac{3}{x^{91}} dx$.

(c) $\int_0^\infty \frac{1}{x^2} dx$.

(d) $\int_7^\infty \frac{2}{x-6}dx.$

(e) $\int_0^\infty e^{-x}dx.$

(f) $\int_0^\infty \frac{1}{x^{2022}+2022^{2022}}dx.$

(g) *Bonus:* $\int_{2023}^\infty \frac{1}{x^{2022}-2022^{2022}}dx.$