Math 199, Fall 2023
Yigal Kamel
9/13/23

Participation assignment 5 - The chain rule

Estimated time: 45-60 minutes.

Point value: 3 points.

Goals: Understand the chain rule and how to use it to compute derivatives.

1) Let $f(x)$ be a function and let $g(x)$ be the inverse function to f, i.e. $f(g(x)) = x$ and $g(f(x)) = x$.

(a) Describe the relationship between the graphs of $f(x)$ and $g(x)$. You can use a sketch in your explanation.

(b) Let a be a number and $b = f(a)$ (notice that this also means $a = g(b)$). Using your answer to (a), describe the relationship between $f'(a)$ and $g'(b)$. Explain in terms of their graphs.

(c) Differentiate both sides of the equation $g(f(x)) = x$ (using the chain rule). Justify your answer to part (b) by plugging in $x = a$ to your result.

2) Apply your result from problem 1 to $f(x) = \ln x$ and $g(x) = e^x$ to deduce the derivative of $\ln(x)$.

3) Apply your result from problem 1 to compute the derivatives of the inverse trigonometric functions. (*Hint:* draw a right triangle with hypotenuse = 1, label an angle θ, and find the lengths of the legs.)

(a) $\dfrac{d}{dx} \arcsin(x)$

(b) $\dfrac{d}{dx} \arccos(x)$

(c) $\dfrac{d}{dx} \arctan(x)$

4) Compute the derivatives of the functions in (a)-(c).

(a) $f(x) = \cos(\cos(\cos x))$

(b) $g(x) = \sqrt{e^{e^x}}$

(c) $h(x) = \ln\left(\dfrac{5x^2 - 2x + 8}{x^3 + x + \frac{2}{x}}\right)$

(d) Calculate $g'(\ln(\ln(4)))$ and $h'(1)$ without a calculator.

5) Recall that a function $f(x)$ is **even** if $f(-x) = f(x)$ and is **odd** if $f(-x) = -f(x)$ for all x.

(a) Give two different examples of even functions and two different examples of odd functions.

(b) What can you say about a function that is both even and odd?

(c) Describe any special properties of the graphs of even and odd functions.

(d) Based on your answer to (c), describe a special property of the derivative $f'(x)$, when f is even, and when f is odd.

(e) Justify your answer to (d) by using the chain rule.