Math 199, Spring 2022
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Participation Assignment 5 - Improper integrals

Estimated Time: 1 hour.

Goals: We want to understand what improper integrals are, how to recognize them, and how to deal with them.

1) What are the two different types of improper integrals?

2) Consider the function $f(x) = \frac{1}{x^2}$.

(a) True or false:

$$\int_{-1}^{1} \frac{1}{x^2} dx = -\frac{1}{x}\Big|_{-1}^{1} = -1 - 1 = -2.$$

(b) Notice that the function $\frac{1}{x^2}$ is always positive where it is defined, but the answer on the right side of the above equation is negative. Is this a problem? Explain why or why not.

(c) Calculate $\int_0^1 \frac{1}{x^2} dx$, or show that it diverges.

(d) Calculate $\int_{-1}^{0} \frac{1}{x^2} dx$, or show that it diverges.

(e) Do your answers satisfy $\int_{-1}^{0} \frac{1}{x^2} dx + \int_{0}^{1} \frac{1}{x^2} dx = -2$?

(f) Explain the collective results for this problem thus far. If your answer to (a) is false, explain the correct way to handle the problem.

3) This problem is related to problem (2) in spirit, but let's treat it as a fresh start.

(a) Calculate $\int_{1}^{\infty} \frac{1}{x^2} dx$, or show that it diverges.

(b) Calculate $\int_{0}^{1} \frac{1}{x^2} dx$, or show that it diverges (you did this already).

(c) Calculate $\int_{1}^{\infty} \frac{1}{\sqrt{x}} dx$, or show that it diverges.

(d) Calculate $\int_0^1 \frac{1}{\sqrt{x}} dx$, or show that it diverges.

(e) Describe a precise relationship between parts (a) and (d) of this problem. Do the same for parts (b) and (c). (*Hint:* turn your head 90 degrees, or interchange the x and y axes.)

4) Determine whether the following improper integrals converge or diverge. Explain why. You do *not* need to find the values of the integrals.

(a) $\int_1^\infty 5dx.$

(b) $\int_4^\infty \frac{3}{x^{91}} dx.$

(c) $\int_0^\infty \frac{1}{x^2} dx.$

(d) $\displaystyle\int_7^\infty \frac{2}{x-6}dx.$

(e) $\displaystyle\int_0^\infty e^{-x}dx.$

(f) $\displaystyle\int_0^\infty \frac{1}{x^{2022}+2022^{2022}}dx.$

(g) *Bonus:* $\displaystyle\int_{2023}^\infty \frac{1}{x^{2022}-2022^{2022}}dx.$