Math 199, Fall 2022
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Participation assignment 2 - Integrals with trig functions

Estimated time: Less than 1 hour (for first two pages).

Point value: 3 points.

Goals: The past two 231 worksheets (and our last 199 prep assignment) were on the method of trigonometric substitution for computing integrals. This method turns an unfamiliar integral into one consisting of trig functions. On this worksheet, we'll focus on practicing integrals with trig functions.

1) Compute the following integrals.

(a) $\int \sin^2 x \cos x dx =$

(b) $\int \cos^3 x dx =$

2) Compute the following integral in two *different* ways.

$\int \sec^2 x \tan x dx =$

3) Your two answers to (2) might look different from each other. Use a trig identity (and your knowledge of integrals) to explain why they are the same.

4) In this problem, I want you to consider some general forms of integrals which involve arbitrary positive integers (i.e. whole numbers) m and n. Explain how you would solve the integral in each general case.

(a) $\int \sin^m x \cos^n x dx$, where $n = 2q + 1$ is odd.
(Hint: try the case where $m = 4$ and $q = 1$.)

(b) $\int \sin^n x dx$, where $n = 2q$ is even.
(Hint: try the case where $q = 2$.)

(c) $\int \sin^n x dx$, where $n = 2q + 1$ is odd.
(Hint: try the case where $q = 2$.)

(d) $\int \sin^m x \cos^n x dx$, where $m = 2p$ and $n = 2q$ are both even.
(Hint: put together ideas from the previous problems.)

(e) *(Bonus:)* $\int \sec^m x \tan^n x dx$.
(Hint: similar to (a)-(d); like those do, you will probably need to separate into cases according to the parity of m and n, i.e. whether they are even or odd.)