

Math 199, Fall 2023

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8/30/23

### Participation assignment 2 - Applying the intermediate value theorem

**Estimated time:** 30-45 minutes.

**Point value:** 2 points.

**Goals:** Understand the ways in which the intermediate value theorem (IVT) can or cannot be applied to solve problems.

1) Explain how the intermediate value theorem can be applied to show that certain continuous functions  $f$  have a root, i.e. a number  $x_0$  such that  $f(x_0) = 0$ .

2) Apply the IVT to show that  $f(x) = 8x^{221} - 30x^{199} - 2023x^{314} + 2$  has a root.

3) Can you *always* use the IVT to determine whether or not a continuous function has a root? Think about what would happen if a function happens to not have a root.

4) Explain how the IVT can be applied to show that two continuous functions  $f$  and  $g$  intersect.

5) Apply the IVT to show that the functions  $f(x) = x^2$  and  $g(x) = 2^x$  intersect more than once.

6) Let 3P% be the percentage of 3-pointers that a basketball player makes. Let's imagine that early on in the 2028 WNBA season, Caitlin Clark's 3P% is 50% and later on in the season it is 41%. Was there necessarily a moment during the season when her 3P% was exactly 45%? Explain how/why the intermediate value theorem does or doesn't apply.

7) Imagine you have an oven whose only settings are off and  $450^\circ\text{F}$ . Use the IVT to argue that it is still possible to use this oven to cook food to an internal temperature of exactly  $250^\circ\text{F}$ .