Math 199, Fall 2023
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Participation assignment 1 - Limits

Estimated time: 50 minutes.

Point value: 3 points.

Goals: Understand limits and how to compute basic examples.

1) Let f be a function and a be a real number.

(a) Explain what the expression $\lim_{x \to a} f(x)$ means in a way that everyone in your group understands.

(b) What is the difference between $\lim_{x \to a} f(x)$ and $f(a)$? Try to give a practical example (that is meaningful to you) of a situation where it is useful to think of these two values as different.

(c) Give an example of a function f and a number a for which $\lim_{x \to a} f(x) = f(a)$, and an example of a function g and a number b for which $\lim_{x \to b} g(x) \neq g(b)$. Explain.

(d) Give an example of a function for which $f(a)$ exists and $\lim_{x \to a} f(x)$ does not. Explain.

(e) Give an example of a function for which $\lim_{x \to a} f(x)$ exists and $f(a)$ does not. Explain.

(f) In you example in (e), can you just define $f(a)$ to be $\lim_{x \to a} f(x)$?

(g) Give examples of two different kinds of situations where $\lim_{x \to a} f(x)$ does not exist, but it is still possible to say something meaningful about what the limit of f at a "is".

(h) Give an example of a situation where $\lim_{x \to a} f(x)$ has no meaning at all.

2) Let $f(x) = \dfrac{x+3}{x-2}$. Compute the following limits.

(a) $\lim_{x \to 0} f(x)$

(b) $\lim_{x \to \infty} f(x)$

(c) $\lim_{x \to 2} f(x)$

(d) $\lim_{x \to 2^-} f(x)$

(e) $\lim_{x \to 2^+} f(x)$

(f) $\lim_{x \to -3} f(x)$

(g) $\lim_{x \to -\infty} f(x)$

(h) Sketch the graph of f.

3) In this problem, we will jump ahead and start doing calculus using limits. Our goal is to compute the slope of the curve $f(x) = x^2$ at the point $x = 1$ using the zooming in process we discussed last time. If we think of h as a small number, then zooming in on the graph of f between $x = 1$ and $x = 1 + h$ will give us a good approximation of the slope.

(a) Explain why the expression $\dfrac{f(x+h) - f(x)}{h}$ is a good approximation for the slope of the graph of f near x.

(b) In order to find the *exact* slope, instead of an approximation, we will take a limit as h gets very small. Find the slope of the graph of f at $x = 1$ by computing $\lim\limits_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$.