Math 199, Fall 2023
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Participation assignment 18 - Volumes via integrals

Estimated time: 40-50 minutes.

Point value: 3 points.

Goals: Practice adapting integration to compute volumes of certain specially selected solids.

1) Recall your soup bowl from last week, whose cross-section takes the shape $y = \frac{1}{3}x^2$ and extends 6 units wide. Find the maximum volume of soup that can fit in the bowl.

2) Derive the formula for the volume of a sphere of radius R by cutting up the sphere into many thin concentric spherical layers. You can use the fact that the surface area of a sphere of radius r is $A = 4\pi r^2$.

3) Derive a formula for the volume of a torus (doughnut) whose cross-sectional circle has radius a and whose central circle has radius b.

4) Derive a formula for the volume of a pyramid with a square base in terms of its side length a and its height h.

5) Consider the region R bounded by the curves $y = e^x$, $x = 1$, $y = 0$, and $x = 3$. Let D be the solid obtained by revolving R around the line $x = -1$. Set up two integrals, one in terms of x and one in terms of y, that compute the volume of D. Compute at least one of these integrals to find the volume of D.

6) A bead is formed from a sphere of radius 5 by drilling a hole through its diameter with a drill bit of radius 2. Set up two different definite integrals which compute the area of the bead (i.e. one "dx" and one "dy"). Find the volume of the bead.