Math 199, Fall 2022
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Participation Assignment 17 - Polar coordinates

Estimated time: 1 hour.

Point value: 3 points.

Goals: Get acquainted with polar coordinates.

Recall that polar coordinates (r, θ) for a point P in the plane are given by:

- $r =$ the distance from the origin to P,
- $\theta =$ the (counter-clockwise) angle from the positive x-axis to the ray from the origin to P.

1) Suppose P is a point in the plane with polar coordinates (r, θ).

(a) Is there a number $\tilde{\theta} \neq \theta$, such that $(r, \tilde{\theta})$ are also polar coordinates for P? If so, what values of $\tilde{\theta}$ allow this? If not, why not?

(b) Is there a number $\tilde{r} \neq r$, such that $(\tilde{r}, \theta)$ are also polar coordinates for P? If so, what values of $\tilde{r}$ allow this? If not, why not?

(c) Are there numbers $\tilde{r} \neq r$ and $\tilde{\theta} \neq \theta$, such that $(\tilde{r}, \tilde{\theta})$ are also polar coordinates for P? If so, what values of $\tilde{r}$ and $\tilde{\theta}$ allow this? If not, why not?

(d) *Bonus:* Is it *ever* possible for $(r, \theta + 1)$ to represent the same point $P = (r, \theta)$?

2) Given an ordinary function $r = f(\theta)$, describe how you would sketch the function by interpreting (r, θ) as polar coordinates. Do the resulting graphs satisfy a property analogous to the "vertical line test"?

3) Try sketching the function $r = \theta$ in polar coordinates, for $0 \leq \theta \leq 4\pi$. Explain why graphing a function in polar coordinates as in (2) is more analogous to a parametric curve, rather than a function $y = f(x)$.

4) In fact, given $r = f(\theta)$, you can always define parametric equations $(x(t), y(t))$ that trace out the polar curve (r, θ). Describe how to do this.

(Note: This sounds fancy, but this is really just about knowing how to convert polar coordinates to rectangular coordinates. The reason I stated the problem this way is to help you recognize that polar curves are simply *examples* of parametric curves, rather than a new thing entirely.)

5) Find a function $r = f(\theta)$ that represents a circle of radius 5 in polar coordinates.

6) Consider the polar curve defined by $r = 3\sec\theta$.

(a) Plug in the values $\theta = 0, \frac{\pi}{4}, \frac{\pi}{3}, -\frac{\pi}{4}$, and plot the corresponding points (r, θ).

(b) Can you guess what the entire curve will be based on these four points?

(c) Let's nail this down algebraically. Recognize that you can rewrite $\sec\theta$ in terms of a more familiar trig function. Then do a little algebra, and convert to rectangular coordinates. Describe the curve.

7) Sketch the polar curves defined by $r_1 = \cos(2\theta)$ and $r_2 = \cos(3\theta)$ on the same graph. Then try to describe what the curve $r = \cos(a\theta)$ would look like for any number a. *Hint:* When is $r = 0$?