

Math 199, Fall 2023
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Participation assignment 15 - Practicing with the FTC

Estimated time: 45-55 minutes.

Point value: 3 points.

Goals: Practice using the fundamental theorem of calculus and get more comfortable with working with definite integrals.

1) Compute the following definite integrals.

(a) $\int_{-2}^0 (2t + 5)dt$

(b) $\int_1^{-1} (r + 1)^2 dr$

(c) $\int_{\pi/2}^{\pi} \frac{\sin(2x)}{2 \sin x} dx$

2) Compute the second derivative $f''(x)$ of the function

$$f(x) = \int_{-e^x}^{2023} t \sin(2t) dt$$

3) Let $f(x) = 2 - \int_2^{x+1} \frac{9}{1+t} dt$. Find the linear approximation of f at the point $x = 1$.

4) Integrals as averages: So far, we've interpreted definite integrals in terms of areas and antiderivatives. The definite integral

$$\int_a^b f(x)dx$$

can also be used to express 'the *average* of the values of $f(x)$ as x ranges from a to b ' = $\text{avg}_a^b f(x)$.

(a) Try to explain how or why this is so.

(b) Find a formula for $\text{avg}_a^b f(x)$.

Hint: Think about the relationship between sums and averages of finitely many values. Thinking about areas can also help. Call me over if you need some extra hints.

5) The speed of a car is $v(t) = 30 - \frac{1}{6}(t - 10)^2 + \sin(3t)$ miles per hour as it completes a 20 minute commute ($t \in [0, 20]$). What is the average speed of the car over the whole commute?

6) The first drop of a roller coaster takes the shape $f(x) = -8(x - 9)^{1/3} + 18$ over the x -interval $[0, 20]$.

(a) Sketch the shape of the drop.

(b) Find the height of the train (units = meters) at the start of the drop, and the height at the bottom of the drop.

(c) Find the average height of the train as it descends the first drop.