

Math 199, Fall 2022
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Participation assignment 15 - Taylor's theorem

Estimated time: 45 minutes.

Point value: 3 points.

Goals: Understand what Taylor's theorem says, and how to use it to better approximate functions by their Taylor polynomials.

1) What is a Taylor polynomial?

2) State Taylor's theorem.

3) Does Taylor's theorem actually tell you what the error is, when approximating a function by a Taylor polynomial?

4) How can you practically use Taylor's theorem to say something about how well a Taylor polynomial approximates a function on a specified interval?

5) How can you use Taylor's theorem to (potentially) show that a function is equal to its Taylor series?

6) Find the second order Taylor polynomial of $f(x) = \tan x$ centered at $x = \pi$. (We previously called this “ $g_2(x)$ ”, and the notes call it $T_2(x)$.)

If we use $T_2(x)$ above to approximate $\tan x$ on the interval $[2.5, 4]$, what is the maximum possible amount our approximation can be wrong by?

How about on the interval $[2, 5]$?

7) Use Taylor’s theorem to prove that the Taylor series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ converges to e^x for every value of x .