Math 199, Fall 2023
Yigal Kamel
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Participation assignment 14 - The fundamental theorem of calculus

Estimated time: 30-50 minutes.

Point value: 3 points.

Goals: Understand the relationship between definite integrals and antiderivatives, and practice applying it to solve problems.

1) Explain what an antiderivative is, and explain what a definite integral is. What is the difference between the two? Are they the same *kind* of object or different kinds of objects (e.g. number, function, shape, angle, arrow, ...)? Explain.

2) One way to summarize the fundamental theorem of calculus is: knowing how to compute definite integrals is equivalent to knowing how to compute antiderivatives.

(a) Given a machine that can compute any antiderivative you want, explain how you can use this machine to compute any definite integral too.

(b) Given a machine that can compute any definite integral you want, explain how you can use this machine to compute any antiderivative too.

3) Does the integral

$$\int_0^{2\pi} \sin x dx$$

represent the area of the region between the graph of $\sin x$ and the x-axis, between the lines $x = 0$ and $x = 2\pi$? Draw a sketch, and either explain why it does, or explain how you can represent the area of the above region in terms of integrals.

4) Compute the integral(s) in problel (3) above, using the FTC, and reevaluate your answer.

5) Using everything you've learned, compute the actual area of the region described above.

6) What is the value of $\int_{-3\pi}^{4\pi} \sin x dx$, just using your answers to (4) and (5).

7) Compute the derivative of the function $f(x) = \int_2^{e^{3x}} e^{2t} dt$.

8) Compute the integral $\int_0^2 2x(x^2 + 7)dx$.

9) However you computed the integral above, can you think of a different way to do it?
Hint: Consider the chain rule, or consider doing algebra.