Math 199, Fall 2022
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11/1/22

Participation assignment 14 - “Hands-on” with Taylor series

Estimated time: 1 hour.

Point value: 3 points.

Goals: The purpose of this worksheet is to better understand the theory of Taylor series and how we can apply that theory in various situations.

1) Recall the definition of the Taylor series of a function $f(x)$ centered at $x = c$. Write down a formula for the Taylor series of $f(x)$ centered at $x = c$.

2) Using a Taylor series of $f(x) = \sqrt{x}$, we'll approximate $\sqrt{101}$ with staggering precision.

(a) By directly calculating the derivatives, find the first four terms (i.e. 0,1,2, and 3) of the Taylor series of $f(x) = \sqrt{x}$ centered at $c = 100$. In other words, find “$g_3(x)$”, as defined in Preparation Assignment 10.

(c) Use the partial sum g_3 to approximate $\sqrt{101}$. In other words, find $g_3(101)$. (Write your answer as a decimal number with 7 terms after the decimal.)

(d) Use a calculator to calculate $\sqrt{101}$, and compare it to your answer to (c).

4) On last Thursday's "Math 231 worksheet", you found a power (Taylor) series for $\tan^{-1}(x)$ by considering the geometric series representation of $\frac{1}{1-x}$.

(a) Using this (you can recalculate it if you need to), find a representation for the number π as a series of rational numbers: i.e.

$$\pi = \sum_{n=0}^{\infty} \frac{p_n}{q_n},$$

where p_n and q_n are integers.

(b) Sum the first six terms of this series to approximate π.

3) Given $f(x) = \sum_{k=0}^{\infty} a_k x^k$ and $g(x) = \sum_{l=0}^{\infty} b_l x^l$, find (your best guess at) the Maclaurin series for $h(x) = f(x)g(x)$. *Hint:* Write $h(x) = \sum_{n=0}^{\infty} c_n x^n$ and find a formula for c_n in terms of the a_k's and b_l's by distributing and collecting like terms. (This formula is important in signal processing.)