Math 199, Spring 2022
Yigal Kamel
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Participation Assignment 13 - Parametric curves and calculus

Estimated Time: 45 minutes.

Goals: Learn to use the techniques of calculus to analyze parametric curves, and become more comfortable with using parametric equations to describe geometrically defined curves.

1) Consider the parametric curve defined by $(x(t), y(t)) = (t^2 - 1, t^3 - t)$.

(a) Calculate the tangent vector $\mathbf{v}(t) = (x'(t), y'(t))$ to the curve for arbitrary t.

(b) Find the values of the curve *and* its tangent vector at the times: $t = 0, \frac{1}{2}, 1, \frac{3}{2}$.

(c) Draw a set of x and y axes, and plot the points and their respective tangent vectors at the times specified in (b).

(d) Using the information the tangent vectors give you about the (infinitesimal) direction of the trajectory, draw an approximate sketch of the curve for all $t \geq 0$. (You can use the same diagram.)

(e) Write down an expression for the speed of the curve at the time t. Then set up an integral that computes the arclength of the portion of the curve that lies below the x-axis.

2) Recall the cycloid example that we did together on Friday. We formed a cycloid by following the path traversed by a point on a wheel rolling on a flat surface. We were able to find parametric equations for the cycloid by separating the path of the center of the wheel from the path of the special point (the "paintbrush") *relative* to the center.

In this problem, use the same method to find parametric equations for an **epicycloid**, the curve traced out by a point on a wheel rolling around another (fixed) circle. To be specific, suppose a (green) circle of radius 2 is rolling counter-clockwise around a fixed (purple) circle of radius 5, starting in the right-most position (as in the figure below). Find parametric equations for the (orange) curve traced out by the point which starts at $(9, 0)$, as pictured.