Math 199, Fall 2023
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Participation assignment 13 - Definite integrals

Estimated time: 25-40 minutes.

Point value: 2 points.

Goals: Practice using finite sums to approximate integrals. Understand what definite integrals are.

1) Consider the function $f(x) = (x+1)(2-x)$.

(a) Sketch the graph of $f(x)$.

(b) Use rectangles of width $\frac{1}{2}$ to approximate the area of the region under the graph and above the x-axis in 4 different ways (make a sketch of the rectangles you are using in each computation):

i. an underestimate (i.e. the area of each rectangle should be less than the area under the actual graph over each interval).

ii. an overestimate (i.e. "... more...").

iii. the left endpoint of each rectangle agrees with the function.

iv. the right endpoint of each rectangle agrees with the function.

(c) We will learn later that (the "fundamental theorem of calculus" says) the area we are interested can be computed as $|F(b) - F(a)|$, where $F(x)$ is an antiderivative of $f(x)$, and a and b are the zeros of $f(x)$. Find the exact area by using this fact.

(d) Compare your answer to (c) with each of your approximations in (b).

(e) Explain why you might expect the recipe in (c) to work by interpreting the area of a single rectangle as " Δ(Area)".

2) What is a definite integral, in your own words?

3) Write down a definition of the definite integral $\int_a^b f(x)dx$.

4) If your answer to (2) used the word "area", give another answer without that word, and explain how it is related to area. If your answer to (2) did not use the word area, explain how/why area can be used to interpret the definite integral.