Math 199, Fall 2022
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Participation assignment 12 - Series of functions

Estimated time: 1 hour.

Point value: 3 points.

Goals: We want to understand series of functions, along with their intervals and radii of convergence.

1) Consider the series (of functions): $\sum_{n=0}^{\infty} \left(\frac{x}{2} + 3\right)^n$.

(a) Write down the (ordinary) series obtained by plugging in $x = 1$. Determine whether the series converges or diverges. If it converges, find the value of the sum.

(b) Write down the (ordinary) series obtained by plugging in $x = -5$. Determine whether the series converges or diverges. If it converges, find the value of the sum.

(c) Write down the (ordinary) series obtained by plugging in $x = -7$. Determine whether the series converges or diverges. If it converges, find the value of the sum.

(d) Write down the (ordinary) series obtained by plugging in $x = -2\pi$. Determine whether the series converges or diverges. If it converges, find the value of the sum.

(e) Find the interval of convergence of the series. (In other words, describe the set of all value of x for which the series converges.)

(f) What is the radius of convergence?

2) Consider the series: $\sum_{n=1}^{\infty} \frac{x^n}{n}$.

(a) You should recognize the series obtained by plugging in $x = 1$ and $x = -1$. Identify these series and state whether each one converges or diverges.

(b) Find the interval of convergence of the series. What is the radius of convergence?

3) Determine all values of x for which the series $\sum_{n=1}^{\infty} \frac{\sin(nx)}{n^2}$ converges.

4) Find the radius and interval of convergence for the series: $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n2^n}$.

5) Find the radius and interval of convergence for the series: $\sum_{n=1}^{\infty} n!x^n$.

6) Find the radius and interval of convergence for the series: $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^{n/2}}$.