Math 199, Fall 2023
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Participation assignment 10 - L'Hôpital's rule

Estimated time: 45-60 minutes.

Point value: 3 points.

Goals: Learn how to identify when L'Hôpital's rule can be used to compute a limit and when it cannot. Learn how to handle situations where an application is not immediate.

1) Suppose $h(x) = \dfrac{f(x)}{g(x)}$. In each of the following cases, state whether or not L'Hôpital's rule can be applied to evaluate $\lim_{x\to a} h(x)$, or if there is not enough information to determine this. Compute the limit in each case you can, or state what additional information is needed. In the cases where L'Hôpital cannot be applied, state whether or not you still have enough information to determine the limit and how to do this.

(a) $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 0$, $\lim_{x\to a} f'(x) = 6$, and $\lim_{x\to a} g'(x) = 2$.

(b) $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = \infty$, $\lim_{x\to a} f'(x) = 5$, and $\lim_{x\to a} g'(x) = \infty$.

(c) $\lim_{x\to a} f(x) = \infty$, $\lim_{x\to a} g(x) = \infty$, $\lim_{x\to a} f'(x)$ DNE, and $\lim_{x\to a} g'(x) = \infty$.

(d) $\lim_{x\to a} f(x) = \infty$, $\lim_{x\to a} g(x) = \infty$, $\lim_{x\to a} f'(x) = \infty$, and $\lim_{x\to a} g'(x) = \infty$.

(e) $\lim_{x\to a} f(x) = \infty$, $\lim_{x\to a} g(x) = 1$, $\lim_{x\to a} f'(x) = 1$, and $\lim_{x\to a} g'(x) = 1$.

(f) $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 1$, $\lim_{x\to a} f'(x) = 9$, and $\lim_{x\to a} g'(x) = 3$.

2) Suppose $h(x) = f(x)g(x)$. Same instructions as problem (1) for the situations below.

(a) $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = 0$, $\lim_{x\to a} f'(x) = 6$, and $\lim_{x\to a} g'(x) = 2$.

(b) $\lim_{x\to a} f(x) = 0$, $\lim_{x\to a} g(x) = \infty$, $\lim_{x\to a} f'(x) = 5$, and $\lim_{x\to a} g'(x) = \infty$.

(c) $\lim_{x\to a} f(x) = \infty$, $g(x) = \frac{f(x)}{x+f(x)^2}$, $\lim_{x\to a} f'(x) = 3$.

3) Let $h(x) = f(x)^{g(x)}$. Same instructions from problems (1) and (2) for the situations below.

(a) $\lim_{x\to a} f(x) = 1$, $\lim_{x\to a} g(x) = \infty$, $\lim_{x\to a} f'(x) = 7$, and $\lim_{x\to a} g'(x) = 11$.

(b) $\lim_{x \to a} f(x) = \infty$, $\lim_{x \to a} g(x) = 0$, $\lim_{x \to a} f'(x) = \infty$, and $\lim_{x \to a} g'(x) = 0$.

4) Evaluate the limits.

(a) $\lim_{t \to \infty} \frac{e^t + t^2}{3e^t - t}$

(b) $\lim_{x \to \infty} \left(\frac{x}{x+2} \right)^x$