Chromatic homotopy theory reading group

Spring 2023

The goal for this reading group is to work towards understanding chromatic homotopy theory, while trying to be accessible to those who are familiar with some (stable) homotopy theory. To facilitate this, we will mostly follow Ravenel's *Nilpotence and periodicity in stable homotopy theory* (Orange book), while drawing from Barthel and Beaudry's *Chromatic structures in stable homotopy theory* (BB) and Ravenel's *Complex cobordism and stable homotopy groups of spheres* (Green book) for modern perspectives and additional topics.

Format: Weekly 1 hour talks from participants.

Meeting information: Wednesday, 10-11am. Room 113 English Building.

Week 0 (1/18). Introduction.

- Green book: Chapter 1.
- BB: Sections 1-2.
- Orange book: 1.4-1.5, 2.4-2.5.

Week 1 (1/25). Review of some homotopy theory.

- Orange book: 1.1-1.3, A.1-A.5, 2.1-2.3.

Week 2 (2/1). Formal group laws and formal groups.

- Green book: Appendix A1.1, A2.
- Include: Hopf algebroids. Moduli stack of formal groups.
- Additional reference: Lurie, Lecture 11.

Week 3 (2/8). MU-theory.

- Orange book: B.1-B.4, 3.1-3.3.

Week 4 (2/15). BP-theory and the Adams spectral sequence.

- Orange book: B.5
- Green book: 4.1-4.3, 2.2.

Week 5 (2/22). Thick subcategories, Landweber exact functor theorem, and Morava K-theories.

- Orange book: 3.4, B.6-B.7, Chapter 5.

Week 6 (3/1). Morava's orbit picture and Morava stabilizer groups.

- Orange book: Chapter 4.
- Additional references: ?

Week 7 (3/8). $K(1)$-local homotopy theory.

- BB: Section 4.

Spring break (3/13-3/17).

Paul Goerss conference (3/20-3/24)

Week 8 (3/29). The periodicity theorem.

- Orange book: Chapter 6.

Week 9 (4/5). Bousfield localization and equivalence.

- Orange book: Chapter 7.
- Additional reference: Ravenel, *Localization with respect to certain homology theories.*

Week 10 (4/12). Localization, smash product, and chromatic convergence theorems.

- Orange book: Chapter 8.

Week 11 (4/19). The nilpotence theorem.

- Orange book: Chapter 9.

Week 12 (4/26). The chromatic fracture square: disassembly and reassembly.

- BB: Sections 2.4, 3.
- Conclude with $L_{K(n)}S^0 \simeq E_n^{h\mathbb{G}_n}$.

Week 13 (5/3). Building pieces of $L_{K(n)}S^0$: higher real K-theories.

- Topics: E_n^{hF} for $F < \mathbb{G}_n$ finite.
- Heard-Stojanoska, *K-theory, reality, and duality*: Section 5.
  - $n = 1$, $F = C_2$, $p = 2$.
  - $n = 1$, $F = C_{p-1}$, $p > 2$.
- Goerss-Henn-Mahowald-Rezk, *A resolution of the K(2)-local sphere at the prime 3*: Section 3.
  - $n = 2$, $F \supseteq C_p$, $p = 3$.
  - $K(2)$-local TMF.

Week 14 (5/10). Finite resolutions of $L_{K(n)}S^0$.

- BB: Section 5.
- Additional reference: Henn *On finite resolutions of $K(n)$-local spheres.*